Question1

Consider the following statements

Assertion (A) : When x, y, z are positive numbers, then

$$\tan^{-1}\left(\sqrt{\frac{x(x+y+z)}{yz}}\right)+\tan^{-1}\left(\sqrt{\frac{y(x+y+z)}{xz}}\right)+\tan^{-1}\left(\sqrt{\frac{z(x+y+z)}{xy}}\right)=\pi$$

Reason (R) : $\tan^{-1} a+\tan^{-1} b=\tan^{-1}\left(\frac{a+b}{1-ab}\right)$, **if** $a>0$ **and** $b>0$

The correct answer is

TG EAPCET 2025 (Online) 2nd May Evening Shift

Options:

A.

Both (A) and (R) are true, (R) is the correct explanation of (A).

B.

Both (A) and (R) are true, (R) is not the correct explanation of (A).

C.

(A) is true, but (R) is false.

D.

(A) is false, but (R) is true.

Answer: C

Solution:

By LHS, $\tan^{-1}\left(\sqrt{\frac{x(x+y+z)}{yz}}\right)+\tan^{-1}\left(\sqrt{\frac{y(x+y+z)}{xz}}\right)+\tan^{-1}\left(\sqrt{\frac{z(x+y+z)}{xy}}\right)$

Let $a=\sqrt{\frac{x(x+y+z)}{yz}}, b=\sqrt{\frac{y(x+y+z)}{xz}}$ and $z=\sqrt{\frac{z(x+y+z)}{xy}}$

Here, $a+b+c=\frac{(x+y+z)^{1/2}(x+y+z)}{\sqrt{xyz}}=\frac{(x+y+z)^{3/2}}{\sqrt{xyz}}$

And $abc=\frac{(x+y+z)^{3/2}}{\sqrt{xyz}}$

Since, $a+b+c=abc$

$\therefore \quad$ LHS $= \pi$

$\Rightarrow$ Assertion is true and since, $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$.

if $xy < 1$

$\Rightarrow$ Reason is false.

Question2

If $e^{\left(\sinh^{-1} 2 + \cosh^{-1} \sqrt{6}\right)} = (a + (b + \sqrt{c})\sqrt{a} + b\sqrt{c})$, then $a + b + c =$

TG EAPCET 2025 (Online) 2nd May Evening Shift

Options:

A.

13

B.

15

C.

17

D.

11

Answer: A

Solution:

We know that

$\sinh^{-1} x = \log\left(x + \sqrt{x^2 + 1}\right)$

$\therefore \sinh^{-1} 2 = \log(2 + \sqrt{4 + 1}) = \log(2 + \sqrt{5})$

$\cosh^{-1} x = \log\left(x + \sqrt{x^2 - 1}\right)$

$\therefore \cosh^{-1} \sqrt{6} = \log(\sqrt{6} + \sqrt{6 - 1}) = \log(\sqrt{6} + \sqrt{5})$

$\therefore \sinh^{-1} 2 + \cosh^{-1} \sqrt{6}$

$= \log(2 + \sqrt{5}) + \log(\sqrt{6} + \sqrt{5})$

$= \log[(2 + \sqrt{5})(\sqrt{6} + \sqrt{5})]$

$e^{\left(\sinh^{-1} \sqrt{2} + \cosh^{-1} \sqrt{6}\right)} = e^{\log((2+\sqrt{5})(\sqrt{6}+\sqrt{5})]}$

$= (2 + \sqrt{5})(\sqrt{6} + \sqrt{5}) = 2\sqrt{6} + 2\sqrt{5} + \sqrt{30} + 5$

$= 5 + 2\sqrt{5} + 2\sqrt{6} + \sqrt{30} = 5 + 2\sqrt{5} + 2\sqrt{6} + \sqrt{5}\sqrt{6}$

$= 5 + (2 + \sqrt{6})\sqrt{5} + 2\sqrt{6} = a + (b + \sqrt{c})\sqrt{a} + b\sqrt{c}$

$\therefore a = 5, b = 2, c = 6$

$\therefore \quad a + b + c = 5 + 2 + 6 = 13$

Question3

Consider the following statements

Assertion (A) For $x \in R - \{1\}$;

$\frac{d}{dx}\left(\tan^{-1}\left(\frac{1+x}{1-x}\right)\right) = \frac{d}{dx}\left(\tan^{-1} x\right)$

Reason (R) For $x < 1, \tan^{-1}\left(\frac{1+x}{1-x}\right) = \frac{\pi}{4} + \tan^{-1} x$, for

$x > 1, \tan^{-1}\left(\frac{1+x}{1-x}\right) = -\frac{3\pi}{4} + \tan^{-1} x$

The correct answer is

TG EAPCET 2025 (Online) 2nd May Evening Shift

Options:

A.

Both (A) and (R) are true, (R) is the correct explanation of (A).

B.

Both (A) and (R) are true, (R) is not the correct explanation of (A).

C.

(A) is true, but (R) is false.

D.

(A) is false, but (R) is true.

Answer: A

Solution:

Given,

$$\frac{d}{dx}\left(\tan^{-1}\left(\frac{1+x}{1-x}\right)\right) = \frac{d}{dx}\left(\tan^{-1} x\right)$$

By LHS, $\frac{d}{dx}\left(\tan^{-1}\left(\frac{1+x}{1-x}\right)\right)$

$$= \frac{1}{1+\left(\frac{1+x}{1-x}\right)^2} \frac{d}{dx}\left(\frac{1+x}{1-x}\right)$$

$$= \frac{(1-x)^2}{(1-x)^2+(1+x)^2}\left[\frac{(1-x)(1)-(1+x)(-1)}{(1-x)^2}\right]$$

$$= \frac{(1-x)^2}{2\left(1+x^2\right)}\left[\frac{1-x+1+x}{(1-x)^2}\right] = \frac{1}{(1+x)^2}$$

And by RHS, $\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$

$\therefore$ Assertion is true.

For reason (R)

when $x<1$, let $x=\tan\theta$

$$\Rightarrow \quad \theta=\tan^{-1} x$$
$$\therefore \quad \tan^{-1}\left(\frac{1+x}{1-x}\right)=\tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$$
$$=\tan^{-1}\left(\tan\left(\frac{\pi}{4}+\theta\right)\right)=\frac{\pi}{4}+\theta$$
$$=\frac{\pi}{4}+\tan^{-1} x$$

When $x>1$, then $\tan^{-1} x>1$

$\therefore \quad \frac{1+x}{1-x}<0$

so, $\tan^{-1}\left(\frac{1+x}{1-x}\right)$ lies in $\left(-\frac{\pi}{2}, 0\right)$

$\therefore \tan^{-1}\left(\frac{1+x}{1-x}\right)=\frac{-3\pi}{4}+\tan^{-1} x$

$\therefore$ Reason is also true.

Question4

If $y=\left(\sin^{-1} x\right)^2$, then $\left(1-x^2\right)\frac{d^2 y}{dx^2}-x\frac{dy}{dx}=$

TG EAPCET 2025 (Online) 2nd May Evening Shift

Options:

A.

$\frac{1}{2}$

B.

2

C.

$-\frac{1}{2}$

D.

4

Answer: B

Solution:

We have, $y=\left(\sin^{-1} x\right)^2$

$$\Rightarrow \quad \frac{dy}{dx}=2\left(\sin^{-1} x\right)\frac{1}{\sqrt{1-x^2}}$$
$$\Rightarrow \quad \sqrt{1-x^2}\frac{dy}{dx}=2\sin^{-1} x$$
$$\Rightarrow \quad \left(1-x^2\right)\left(\frac{dy}{dx}\right)^2=4\left(\sin^{-1} x\right)^2$$
$$\Rightarrow \quad \left(1-x^2\right)\left(\frac{dy}{dx}\right)^2=4y \text{ (Using Eq.}$$

Again, differentiating w.r.t x, we get

$$(-2x)\left(\frac{dy}{dx}\right)^2 + (1-x^2)\left(2\frac{dy}{dx}\right)\frac{d^2y}{dx^2} = 4\frac{dy}{dx}$$
$$\Rightarrow (-2x)\frac{dy}{dx} + 2(1-x^2)\frac{d^2y}{dx^2} = 4$$
$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$$

Question5

The range of the real value function $f(x) = \sin^{-1}\left(\sqrt{x^2+x+1}\right)$ is

TG EAPCET 2025 (Online) 2nd May Morning Shift

Options:

A. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

B. $\left[0, \frac{\pi}{2}\right]$

C. $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$

D. $\left[\frac{\pi}{3}, \frac{\pi}{2}\right]$

Answer: D

Solution:

Given, $f(x) = \sin^{-1}\left(\sqrt{x^2+x+1}\right)$

The domain of $\sin^{-1}\left(\sqrt{x^2+x+1}\right)$ is $[-1, 1]$

$\therefore \quad -1 \leq \sqrt{x^2+x+1} \leq 1$

Since $\sqrt{x^2+x+1}$ is always,

non-negative, so $0 \leq \sqrt{x^2+x+1} \leq 1$

Let $g(x) = x^2+x+1 = \left(x+\frac{1}{2}\right)^2 + \frac{3}{4}$

The minimum value of $g(x)$ is $3/4$, which occurs at $x = -\frac{1}{2}$

$\therefore \quad x^2+x+1 \geq \frac{3}{4}$

$\Rightarrow \quad \sqrt{x^2+x+1} \geq \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \frac{\sqrt{3}}{2} \leq \sqrt{x^2+x+1} \leq 1$

$\Rightarrow \quad \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \leq \sin^{-1}\left(\sqrt{x^2+x+1}\right)$

$\Rightarrow \quad \frac{\pi}{3} \leq f(x) \leq \frac{\pi}{2} \quad \leq \sin^{-1}$

Thus, the range of $f(x) = \sin^{-1}\left(\sqrt{x^2+x+1}\right)$ is $\left[\frac{\pi}{3}, \frac{\pi}{2}\right]$

Question6

$\tan^{-1}\frac{3}{5}+\tan^{-1}\frac{6}{41}+\tan^{-1}\frac{9}{191}=$

TG EAPCET 2025 (Online) 2nd May Morning Shift

Options:

A.

$\tan^{-1}\frac{9}{10}$

B.

$\tan^{-1}\frac{18}{19}$

C.

$\tan^{-1}\frac{3}{191}$

D.

$\tan^{-1}\frac{6}{205}$

Answer: A

Solution:

$$\begin{aligned}&\tan\left(\frac{3}{5}\right)+\tan^{-1}\left(\frac{6}{41}\right)\\&=\tan^{-1}\left(\frac{\frac{3}{5}+\frac{6}{41}}{1-\frac{3}{5}\cdot\frac{6}{41}}\right)=\tan^{-1}\left(\frac{\frac{153}{205}}{\frac{187}{205}}\right)\\&=\tan^{-1}\left(\frac{153}{187}\right)\end{aligned}$$

$$\begin{aligned}&\text{Now, }\tan^{-1}\left(\frac{153}{187}\right)+\tan^{-1}\left(\frac{9}{191}\right)\\&=\tan^{-1}\left(\frac{\frac{153}{187}+\frac{9}{191}}{1-\frac{153}{187}\cdot\frac{9}{191}}\right)\\&=\tan^{-1}\left(\frac{\frac{29223+1683}{35717}}{1-\frac{1377}{35717}}\right)\\&=\tan^{-1}\left(\frac{30906}{34340}\right)=\tan^{-1}\left(\frac{9}{10}\right)\end{aligned}$$

Question7

If $2\tanh^{-1}x=\sinh^{-1}\left(\frac{4}{3}\right)$, then $\cosh^{-1}\left(\frac{1}{x}\right)=$

TG EAPCET 2025 (Online) 2nd May Morning Shift

Options:

A.

$\log(\sqrt{2}+1)$

B.

$\log(\sqrt{2}-1)$

C.

$\log(2+\sqrt{3})$

D.

$\log(2-\sqrt{3})$

Answer: C

Solution:

$$\text{Given, } 2\tanh^{-1}x = \sinh^{-1}\left(\frac{4}{3}\right)$$
$$\tanh^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$
$$\Rightarrow \quad 2\tanh^{-1}(x) = \ln\left(\frac{1+x}{1-x}\right)$$
$$\text{So, } \ln\left(\frac{1+x}{1-x}\right) = \sinh^{-1}\left(\frac{4}{3}\right)$$
$$= \ln\left(\frac{4}{3}+\sqrt{\left(\frac{4}{3}\right)^2+1}\right)$$
$$\left[\because \sinh^{-1}(y) = \ln\left(y+\sqrt{y^2+1}\right)\right]$$
$$= \ln\left(\frac{4}{3}+\sqrt{\frac{16}{9}+1}\right)$$
$$= \ln\left(\frac{4}{3}+\frac{5}{3}\right) \Rightarrow \ln\frac{9}{3} = \ln(3)$$
$$= \frac{1+x}{1-x} = 3 \Rightarrow 1+x = 3-3x$$
$$= 4x = 2 \Rightarrow x = \frac{1}{2}$$
$$\text{Now, } \cosh^{-1}\left(\frac{1}{x}\right) = \cosh^{-1}(4$$
$$= \ln(2+\sqrt{4-1})$$
$$\left[\because \cosh^{-1}(z) = \ln\left(z+\sqrt{z^2-1}\right)\right]$$
$$= \ln(2+\sqrt{3})$$

Question8

If $f(x) = \sqrt{\cos^{-1}\sqrt{1-x^2}}$, then $f'\left(\frac{1}{2}\right) =$

TG EAPCET 2025 (Online) 2nd May Morning Shift

Options:

A.

$\sqrt{\frac{2}{\pi}}$

B.

$\sqrt{\frac{\pi}{2}}$

C.

$-\sqrt{\frac{2}{\pi}}$

D.

$-\sqrt{\frac{\pi}{2}}$

Answer: A

Solution:

Given, $f(x)=\sqrt{\cos^{-1}\sqrt{1-x^2}}$

Let $x=\sin\theta$.

Then, $\sqrt{1-x^2}=\sqrt{1-\sin^2\theta}$

$\Rightarrow \quad \sqrt{\cos^2\theta}=|\cos\theta|$

Since, $\theta\in\left[0,\frac{\pi}{2}\right]$, so $\cos\theta\geq 0$

$\therefore \quad \sqrt{1-x^2}=\cos\theta$

So, $f(x)=\sqrt{\cos^{-1}(\cos\theta)}$

$$\Rightarrow \quad \sqrt{\theta}=\sqrt{\sin^{-1}(x)}$$

$$f'(x)=\frac{d}{dx}\sqrt{\sin^{-1}(x)}$$

$$\Rightarrow \quad \frac{1}{\sqrt{1-x^2}}\cdot\frac{1}{2\sqrt{\sin^{-1}(x)}}$$

$$f'(x)=\frac{1}{\sqrt{1-\left(\frac{1}{2}\right)^2}}\cdot\frac{1}{2\cdot\sqrt{\sin^{-1}\frac{1}{2}}}$$

$$=\frac{1}{\frac{\sqrt{3}}{2}}\cdot\frac{1}{2\cdot\sqrt{\frac{\pi}{6}}}$$

$$\Rightarrow \quad \frac{2}{\sqrt{3}}\cdot\frac{\sqrt{6}}{2\sqrt{\pi}}\Rightarrow\frac{\sqrt{2}}{\sqrt{\pi}}=\sqrt{\frac{2}{\pi}}$$

$$\therefore \quad f'(x)=\sqrt{\frac{2}{\pi}}$$

Question9

If $\sin^{-1}x-\cos^{-1}2x=\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)-\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$**, then** $\tan^{-1}x+\tan^{-1}\left(\frac{x}{x+1}\right)=$

TG EAPCET 2024 (Online) 11th May Morning Shift

Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: B

Solution:

$\sin^{-1} x - \cos^{-1} 2x = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$

$\Rightarrow \frac{\pi}{2} - \cos^{-1} x - \cos^{-1} 2x = \frac{\pi}{6}$

$\Rightarrow \cos^{-1} x + \cos^{-1} 2x = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$

$\Rightarrow \cos^{-1}\left(x \cdot 2x - \sqrt{1-x^2}\sqrt{1-4x^2}\right) = \frac{\pi}{3}$

$\Rightarrow 2x^2 - \sqrt{1-x^2}\sqrt{1-4x^2} = \cos\frac{\pi}{3} = \frac{1}{2}$

$\Rightarrow \sqrt{1-x^2}\sqrt{1-4x^2} = \left(2x^2 - \frac{1}{2}\right)^2$

On solving, we get

$x = \frac{1}{2}$

$\tan^{-1} x + \tan^{-1}\left(\frac{x}{x+1}\right) = \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}$

$= \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{6}}\right)$

$= \tan^{-1}\left(\frac{\frac{5}{6}}{\frac{5}{6}}\right) = \tan^{-1} 1 = \frac{\pi}{4}$

Question10

$\operatorname{sech}^{-1}\left(\frac{3}{5}\right) - \tanh^{-1}\left(\frac{3}{5}\right) =$

TG EAPCET 2024 (Online) 11th May Morning Shift

Options:

A. $\log_e 6$

B. $\log_e 5$

C. $\log_e\left(\frac{3}{2}\right)$

D. $\log_e\left(\frac{2}{3}\right)$

Answer: C

Solution:

$\sec h^{-1}\frac{3}{5} - \tan^{-1}\frac{3}{5}$

Let $\operatorname{sech}^{-1}\frac{3}{5} = x$ and $\tan h^{-1}\frac{3}{5} = y$

$\sec hx = \frac{3}{5}$ and $\tan hy = \frac{3}{5}$

$x = \ln 3 \quad e^{2y} = 4$

$ey = 2$

$y = \ln 2$

Now, $x - y = \ln 3 - \ln 2 = \ln \frac{3}{2}$

Question11

The domain of the real valued function $f(x) = \sin^{-1}\left(\log_2\left(\frac{x^2}{2}\right)\right)$ is

TG EAPCET 2024 (Online) 10th May Evening Shift

Options:

A. $[-2, 0) \cup (1, 2]$

B. $[-2, -1] \cup [1, 2]$

C. $[-1, 0] \cup [1, 2]$

D. $[1, \infty) \cup (-2, 0)$

Answer: B

Solution:

To find the domain of the function $f(x) = \sin^{-1}\left(\log_2\left(\frac{x^2}{2}\right)\right)$, we need to determine where the expression $\log_2\left(\frac{x^2}{2}\right)$ falls within the domain of the inverse sine function, which is $[-1, 1]$.

First, ensure that:

$-1 \leq \log_2\left(\frac{x^2}{2}\right) \leq 1$

This inequality translates to:

$2^{-1} \leq \frac{x^2}{2} \leq 2$

Solving the inequality:

$\frac{1}{2} \leq \frac{x^2}{2} \leq 2$

Multiply through by 2 to clear the fraction:

$1 \leq x^2 \leq 4$

This implies that x must satisfy:

$x \in [-2, -1] \cup [1, 2]$

We also need to ensure that $\frac{x^2}{2} \geq 0$, which is always true since $x^2 \geq 0$.

Therefore, considering the intersection of the feasible solutions:

$x \in [-2, -1] \cup [1, 2]$

Thus, the domain of the function is:

$x \in [-2, -1] \cup [1, 2]$

Question12

The trigonometric equation $\sin^{-1} x = 2\sin^{-1} a$, has a solution

TG EAPCET 2024 (Online) 10th May Evening Shift

Options:

A. only when $\frac{1}{\sqrt{2}} < a < \frac{1}{2}$

B. for all real values of (a)

C. only when $|a| \leq \frac{1}{\sqrt{2}}$

D. only when $|a| \geq \frac{1}{\sqrt{2}}$

Answer: C

Solution:

To solve the trigonometric equation $\sin^{-1} x = 2\sin^{-1} a$, we must consider the following:

Given that $\sin^{-1} x = 2\sin^{-1} a$, it implies:

$\sin^{-1} x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

This corresponds to:

$2\sin^{-1} a \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

From this, we can deduce:

$\sin^{-1} a \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

Consequently, the values for a must satisfy:

$a \in \left[-\sin\frac{\pi}{4}, \sin\frac{\pi}{4}\right]$

This simplifies to:

$a \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

Therefore, the condition for a is:

$|a| \leq \frac{1}{\sqrt{2}}$

Question13

If the real valued function $f(x) = \sin^{-1}\left(x^2 - 1\right) - 3\log_3\left(3^x - 2\right)$ is not defined for all $x \in (-\infty, a) \cup (b, \infty)$, then $3^a + b^2 =$

TG EAPCET 2024 (Online) 10th May Morning Shift

Options:

A. 5

B. 6

C. 3

D. 4

Answer: D

Solution:

Given,

$f(x)=\sin^{-1}\left(x^2-1\right)-3\log_3\left(3^x-2\right)$

$$-1 \leq x^2-1 \leq 1$$

$$0 \leq x^2 \leq 2$$

$$-\sqrt{2}<x<\sqrt{2}$$

$\therefore \sin^{-1}\left(x^2-1\right)$ is not defined for $(-\infty,-\sqrt{2})$ and $(\sqrt{2}, 0)$

For $3\log_3\left(3^x-2\right) \quad \Rightarrow \quad 3^x-2>0$

$3^x>2$

$x>\log_3 2$

$\therefore 3\log_3\left(3^x-2\right)$ is not defined for $x \leq \log_3(2)$

So, intervel $[-\sqrt{2}, \sqrt{2}]$ combine with $x>\log_3(2)$. The domain of $f(x)$ is $\log _3(2)

So, $f(x)$ is not defined for

$(-\infty, \log_3 2) \cup(\sqrt{2}, 0)$

$a=\log_3 2, b=\sqrt{2}$

$3^a+b^2=2+2=4$

Question14

If $\sin^{-1}(4x)-\cos^{-1}(3x)=\frac{\pi}{6}$, then $x=$

TG EAPCET 2024 (Online) 10th May Morning Shift

Options:

A. $\frac{\sqrt{3}}{2\sqrt{7}}$

B. $\frac{\sqrt{3}}{4\sqrt{7}}$

C. $\frac{\sqrt{3}}{2\sqrt{13}}$

D. $\frac{\sqrt{3}}{4\sqrt{13}}$

Answer: C

Solution:

Given, $\sin^{-1}(4x) - \cos^{-1}(3x) = \frac{\pi}{6}$

$\Rightarrow \frac{\pi}{2} - \cos^{-1} 4x - \cos^{-1} 3x = \frac{\pi}{6}$

$\Rightarrow \cos^{-1} 4x + \cos^{-1} 3x = \frac{\pi}{3}$

$\Rightarrow 4x \times 3x - \sqrt{1-16x^2}\sqrt{1-9x^2} = \cos\frac{\pi}{3}$

$\Rightarrow \left(12x^2 - \frac{1}{2}\right)^2 = \left(1-16x^2\right)\left(1-9x^2\right)$

$\Rightarrow 144x^4 + \frac{1}{4} - 12x^2 = 1 - 25x^2 + 144x^4$

$13x^2 = \frac{3}{4} \Rightarrow x^2 = \frac{3}{4\times 13} \Rightarrow x = \frac{\sqrt{3}}{2\sqrt{13}}$

Question15

If $\sin h^{-1}(-\sqrt{3}) + \cos^{-1}(2) = K$**, then** $\cosh K =$

TG EAPCET 2024 (Online) 10th May Morning Shift

Options:

A. $\log(2-\sqrt{3})$

B. $\log(2+\sqrt{3})$

C. 0

D. 1

Answer: D

Solution:

Given,

$\sinh^{-1}(-\sqrt{3}) + \cosh^{-1}(2) = K$

$\log(-\sqrt{3} + \sqrt{3+1}) + \log(2+\sqrt{3}) = K$

$\Rightarrow K = \log[(2-\sqrt{3})(2+\sqrt{3})]$

$\Rightarrow K = \log 1 = 0$

$\cosh K = \cosh 0 = \frac{e^0 + e^{-0}}{2} = 1$

Question16

If $y = \cos^{-1}\left(\frac{6x-2x^2-4}{2x^2-6x+5}\right)$**, then** $\frac{dy}{dx} =$

TG EAPCET 2024 (Online) 10th May Morning Shift

Options:

A. $\frac{2}{\sqrt{3x-x^2-2}}$

B. $\frac{2}{3x-x^2-2}$

C. $\frac{2}{\sqrt{2x^2-6x+5}}$

D. $\frac{2}{2x^2-6x+5}$

Answer: D

Solution:

$y=\cos^{-1}\left(\frac{-2x^2+6x-4}{2x^2-6x+5}\right)$

$\Rightarrow \frac{dy}{dx}=\frac{-1}{\sqrt{1-\left(\frac{6x-2x^2-4}{2x^2-6x+5}\right)^2}}$

$\Rightarrow \frac{dy}{dx}=\frac{-1}{\sqrt{\frac{\left(2x^2-6x+5\right)^2-\left(-\left(2x^2-6x+4\right)^2\right.}{\left(2x^2-6x+5\right)}}}$

$\frac{\left(12x^2-36x+30-8x^3+24x^2-20x\right)}{-\left(24x^2-8x^3-16x-36x+12x^2+24\right.} \frac{\left(2x^2-6x+5\right)^2}{}$

$\Rightarrow \frac{dy}{dx}=\frac{-1\left(2x^2-6x+5\right)}{\sqrt{4x^2-12x+9}}.$

$12x^2-36x+30-8x^3+24x^2-20x$

$-24x^2+8x^3+16x+36x-12x-24$

$\Rightarrow \frac{dy}{dx}=\frac{-1\left(2x^2-6x+5\right)^2}{\sqrt{4x^2-12x+9}}\cdot\frac{(-4x+6}{\left(2x^2-6x+5\right)^2}$

$\Rightarrow \frac{dy}{dx}=\frac{2(2x-3)}{(2x-3)(2x^2-6x+5)}$

$\Rightarrow \frac{dy}{dx}=\frac{2}{2x^2-6x+5}$

Question17

If $2\tan^{-1}x=3\sin^{-1}x$ and $x\neq 0$, then $8x^2+1=$

TG EAPCET 2024 (Online) 9th May Evening Shift

Options:

A. 13

B. 5

C. $\sqrt{7}$

D. $\sqrt{17}$

Answer: D

Solution:

Given, $2\tan^{-1}x = 3\sin^{-1}x$ and $x \neq 0$

$\therefore \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \sin^{-1}\left(3x - 4x^3\right)$

$\Rightarrow \quad \frac{2x}{1+x^2} = x\left(3 - 4x^2\right)$

$\Rightarrow \quad 2 = \left(1 + x^2\right)\left(3 - 4x^2\right) \quad [\because x \neq 0]$

$\Rightarrow \quad 4x^4 + x^2 - 1 = 0$

$\Rightarrow \quad x^2 = \frac{-1+\sqrt{(1)^2-4(4)(-1)}}{2(4)} [\because 0 < x^2]$

$\Rightarrow \quad 8x^2 = -1 + \sqrt{17} \Rightarrow 8x^2 + 1 = \sqrt{17}$

Question18

Match the functions given in List I with their relevant characteristics from List II.

List I	List II
(A) sinh x	(I) Domains is (-1,1), even function
(B) sec hx	(II) Domain is [1,∞), neither even nor odd function
(C) tan hx	(III) Even function
(D) cosec h^{-1}x	(IV) Range is R, odd function
	(V) Range is (-1,1), odd function

The correct answer is

TG EAPCET 2024 (Online) 9th May Evening Shift

Options:

A. A-II, B-III, C-IV, D-V

B. A-V, B-I, C-II, D-III

C. A-IV, B-II, C-I, D-V

D. A-IV, B-III, C-V, D-II

Answer: D

Solution:

(a) $\sinh x$ is an odd function and its range is R and domain is R.

(b) $\operatorname{sech} x$ is an even function and its range is (0,1] and domain is R.

(c) $\tanh x$ is an odd function and its range is $(-1, 1)$ and domain is R.

(d) $\operatorname{cosec}^{-1} x$ is neither even nor odd function.

Hence, A-IV, B-III, C-V, D-II

Question19

$\cos^{-1}\frac{3}{5}+\sin^{-1}\frac{5}{13}+\tan^{-1}\frac{16}{63}=$

TG EAPCET 2024 (Online) 9th May Morning Shift

Options:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{6}$

Answer: A

Solution:

We have,

$\cos^{-1}\frac{3}{5}+\sin^{-1}\frac{5}{13}+\tan^{-1}\frac{16}{63}$

$=\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{5}{12}+\tan^{-1}\frac{16}{63}$

$\left[\because \cos^{-1}\frac{3}{5}=\tan^{-1}\frac{4}{3} \text{ and } \sin^{-1}\frac{5}{13}=\tan^{-1}\frac{5}{12}\right]$

$=\tan^{-1}\frac{63}{16}+\tan^{-1}\frac{16}{63}$

$=\cot^{-1}\frac{16}{63}+\tan^{-1}\frac{16}{63}=\frac{\pi}{2}$

Question20

If $\cosh^{-1}\left(\frac{5}{3}\right)+\sinh^{-1}\left(\frac{3}{4}\right)=k$**, then** $e^{k}=$

TG EAPCET 2024 (Online) 9th May Morning Shift

Options:

A. $\frac{2}{3}$

B. $\frac{3}{2}$

C. 6

D. 5

Answer: C

Solution:

We have,

$$\cosh^{-1}\left(\frac{5}{3}\right)+\sinh^{-1}\left(\frac{3}{4}\right)=k$$

$$\ln\left(\frac{5}{3}+\sqrt{\left(\frac{5}{3}\right)^{2}-1}\right)+\ln\left[\left(\frac{3}{4}\right)+\sqrt{\left(\frac{3}{4}\right)^{2}+1}\right]=k$$

$$\left[\begin{array}{c}\because \cos h^{-1}x=\ln\left(x+\sqrt{x^{2}-1}\right)\\ \text{and } \sin h^{-1}x=\ln\left(x+\sqrt{x^{2}+1}\right)\end{array}\right]$$

$$\ln\left(\frac{5}{3}+\frac{4}{3}\right)+\ln\left(\frac{3}{4}+\frac{5}{4}\right)=k$$

$$\ln 3+\ln 2=k$$

$$\ln 6=k$$

$$e^{k}=6$$
